**5.7 Notes and Examples** Name: Block: Seat:

*Inverse Trig Derivatives: Derivatives*

1. Warm up: Trig Drill. The sine function takes an angle, and returns a ratio. The arcsine function takes

   a ______________, and returns an ______________

   (a) If $\sin \frac{\pi}{6} = \frac{1}{2}$, then $\arcsin \frac{1}{2} =$

   (b) If $\sin -\frac{\pi}{6} = -\frac{1}{2}$, then $\arcsin -\frac{1}{2} =$

   (c) If $\sin \frac{5\pi}{6} = \frac{1}{2}$, then $\arcsin \frac{1}{2} =$

   (d) If $\sin \frac{11\pi}{6} = -\frac{1}{2}$, then $\arcsin -\frac{1}{2} =$

   (e) Try practicing here: `https://www.mathorama.com/trigdrill/`
   (Press the red $\boxed{\text{Start}}$ button to begin)

2. Review of arcsine ($\arcsin x, \sin^{-1} x$), arccosine ($\arccos x, \cos^{-1} x$), and arctangent ($\arctan x, \tan^{-1} x$)

   (a) In order to have inverse functions of periodic trig functions, we restrict the range.

   (b) Arcsine and Arctangent: If the ratio is ______________, the angle returned is

   between ______________ and ______________ (Quadrant ____)

   (c) Arcsine and Arctangent: If the ratio is ______________, the angle returned is

   between ______________ and ______________ (Quadrant ____)

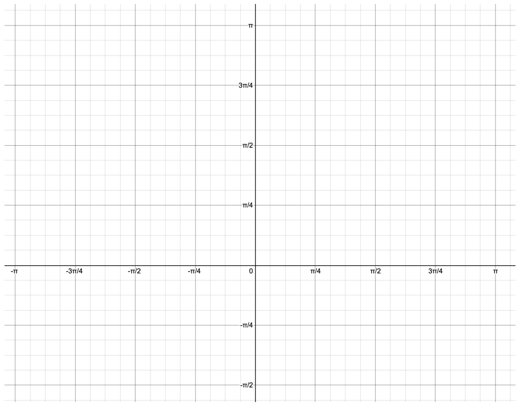
   (d) Arccosine: If the ratio is ______________, the angle returned is

   between ______________ and ______________ (Quadrant ____)

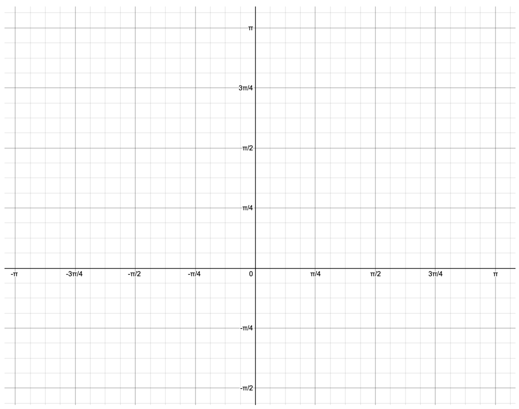
   (e) Arccosine: If the ratio is ______________, the angle returned is

   between ______________ and ______________ (Quadrant ____)

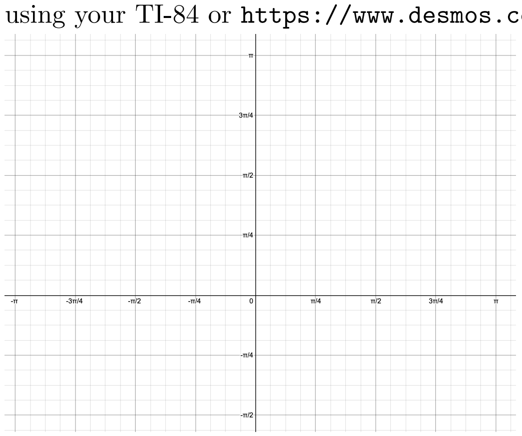
(f) Graph $y = \arcsin x$ and the restricted $y = \sin x\{-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\}$
using your TI-84 or `https://www.desmos.com/calculator/sxjpz2cb63`



(g) Graph $y = \arctan x$ and the restricted $y = \tan x\{-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\}$
using your TI-84 or `https://www.desmos.com/calculator/sxjpz2cb63`



(h) Graph $y = \arccos x$ and the restricted $y = \cos x\{0 \leq x \leq \pi\}$
using your TI-84 or `https://www.desmos.com/calculator/sxjpz2cb63`

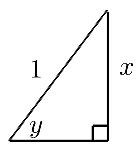


(i) The domain of $\arcsin x$ is ______________

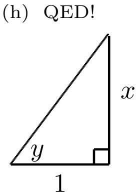
(j) The domain of $\arccos x$ is ______________

(k) The domain of $\arctan x$ is ______________

3. Deriving the Derivative of $y = \arcsin x$
   (a) Draw a right triangle with hypotenuse 1, acute angle $y$ and opposite leg with length $x$.
   (b) Use the Pythagorean Theorem to find the length of the adjacent leg.
   (c) Start with $y = \arcsin x$ (note how this true from our drawing)
   (d) Take the sine of both sides (note how this can also be verified from the drawing SOH-CAH-TOA)
   (e) Implicitly differentiate both sides with respect to $x$ (Remember the chain rule)
   (f) Divide both sides by $\cos y$
   (g) Substitute $\cos y$ with "adjacent over hypotenuse" from the drawing.
   (h) QED!




4. Deriving the Derivative of $y = \arctan x$
   (a) Draw a right triangle with adjacent leg 1, acute angle $y$ and opposite leg with length $x$.
   (b) Use the Pythagorean Theorem to find the length of the hyptenuse.
   (c) Start with $y = \arctan x$ (note how this true in our drawing)
   (d) Take the tangent of both sides (note how this can also be verified from the drawing using SOH-CAH-TOA)
   (e) Implicitly differentiate both sides with respect to $x$ (Remember the chain rule)
   (f) Substitute $\sec^2 y$ with "hypotenuse over adjacent squared" from the drawing.
   (g) Solve for $\frac{dy}{dx}$
   (h) QED!




Try deriving $y = \arccos x$ on your own, or if you need help: `https://www.mathorama.com/gsp/Arccosine.pdf`

**The Theorems**

$$\frac{d}{dx}\left[\arcsin x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}}\ dx =$$

If $u$ is differentiable function of $x$:

$$\frac{d}{dx}\left[\arcsin u\right] = \frac{u'}{\sqrt{1-u^2}} \qquad \int \frac{u'}{\sqrt{1-u^2}}\ dx =$$

$$\frac{d}{dx}\left[\arctan x\right] = \frac{1}{1+x^2} \qquad \int \frac{1}{1+x^2}\ dx =$$

If $u$ is differentiable function of $x$:

$$\frac{d}{dx}\left[\arctan u\right] = \frac{u'}{1+u^2} \qquad \int \frac{u'}{1+u^2}\ dx =$$

1. Examples
   (a) If $f(x) = \arcsin(2x)$, find $f'(x)$

   (b) If $f(x) = \arctan(3x)$, find $f'(x)$

   (c) If $f(x) = \arcsin\sqrt{x}$, find $f'(x)$

   (d) If $f(x) = \arcsin x + x\sqrt{1-x^2}$, find $f'(x)$